Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Differential and Integral Calculus

Subject Code: 4SC04DIC1 **Branch: B.Sc. (Mathematics, Physics)**

Semester: 4 **Date: 18/04/2019** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: **(14)**

a) Define: Gradient (02)

b) If $\bar{F} = grad\,\phi$ then $curl\,\bar{F} =$ ______. (01)

c) What is the value of $div(\phi u)$, if u is a vector point function and ϕ is a scalar point function? (01)

d) Evaluate: $\int_0^2\int_0^2 (x^2+y^2)\,dy\,dx$ (02)

e) Evaluate: $\int_1^2\int_0^a\int_2^4 xy^2z^3\,dz\,dy\,dx$ (02)

f) State Stoke's theorem. (02)

g) Define: Curvature (02)

h) Explain in short Lagrange's equation for partial differential equation. (02)

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions **(14)**

a) Find the directional derivative of $\phi = (x^2+y^2+z^2)^{-\frac{1}{2}}$ at the point $(3,1,2)$ in the direction of the vector $yz\hat{i}+xz\hat{j}+xy\hat{k}$. (05)

b) Evaluate $\int_C \bar{F}\cdot d\bar{r}$, where $\bar{F} = x^2\,\hat{i}+xy\,\hat{j}$ and C is the boundary of the square in the plane $z=0$ and bounded by the lines $x=0, y=0, x=a$ and $y=a$. (05)

c) Determine the constants a and b such that $\bar{A}$ is irrotational, where $\bar{A} = (2xy+3yz)\hat{i}+(x^2+axz-4z^2)\hat{j}+(3xy+2byz)\hat{k}$. (04)

Q-3 Attempt all questions **(14)**

a) If $\bar{r} = x\hat{i}+y\,\hat{j}+z\,\hat{k}$ then show that $div(grad(r^n)) = n(n+1)r^{n-2}$. (05)

b) Evaluate $\iint_A x^2\,dx\,dy$ where A is the region in the first quadrant bounded by the hyperbola $xy = 16$ and the lines $x = y, y = 0, x = 8$. (05)

c) Find the equation of tangent plane and normal line to the surface $xyz = 6$ at the point $(1,2,3)$. (04)

Q-4 Attempt all questions **(14)**

a) If u, v are vector point functions and ϕ is a scalar point function then prove that $div(u \times v) = (curl u)\cdot v - u\cdot(curl v)$. (07)

b) Evaluate $\iint_S \bar{F}\cdot\hat{n}\,dS$, where $\bar{F} = 3y\hat{i} + 2z\hat{j} + x^2yz\hat{k}$ and S is the surface $y^2 = 5x$ in the positive octant bounded by the plane $x = 3, z = 4$. (05)

c) Evaluate: $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_0^{1-2\cos^2\theta} \int_0^1 r\sin\theta\,dz\,dr\,d\theta$ (02)

Q-5 Attempt all questions **(14)**

a) State and prove Green's theorem. (07)

b) Evaluate $\iint_R (x+y)\,dA$, where R is Trapezoidal region with vertices $(0,0), (5,0)$, $\left(\frac{5}{2},\frac{5}{2}\right), \left(\frac{5}{2},-\frac{5}{2}\right)$ by using transformation $x = 2u + 3v$ and $y = 2u - 3v$. (05)

c) State Gauss-divergence theorem. (02)

Q-6 Attempt all questions **(14)**

a) Find $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2 + y^2)\,dy\,dx$ by changing into polar co-ordinates. (05)

b) Evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$ by change the order of integration. (05)

c) Find the work done when a force $\bar{F} = (x^2 - y^2 + x)\hat{i} + (2x + y)\hat{j}$ moves a particle from origin to $(1,1)$ along a parabola $y^2 = x$. (04)

Q-7 Attempt all questions **(14)**

a) Verify Gauss-divergence theorem for $\bar{F} = 2xz\hat{i} + yz\hat{j} + z^2\hat{k}$ for the upper half sphere $x^2 + y^2 + z^2 = a^2$. (10)

b) Find the radius of curvature at any point on the curve $y^2 = 4ax$. (04)

Q-8 Attempt all questions **(14)**

a) Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \cos y$, given that $\frac{\partial z}{\partial y} = -2\cos y$ when $x = 0$ and $z = 0$ when y is a multiple of π. (04)

b) Form the partial differential equation by eliminating the arbitrary function from $z = xy + f\left(x^2 + y^2\right)$. (03)

c) Show that the radius of curvature at any point on the cardioids $r = a\left(1 - \cos\theta\right)$ is $\frac{2}{3}\sqrt{2ar}$. (04)

d) Solve: $\frac{y^2 z}{x}\frac{\partial z}{\partial x} + xz\frac{\partial z}{\partial y} = y^2$ (03)